

Completing the Square

Complete the square to get the following quadratic expressions in the form $p(x+q)^2 + r$.

$$\begin{aligned}
 (1) \quad x^2 + 12x - 10 &= \left(x^2 + 12x + \frac{36}{1} \right) - \frac{36}{1} - 10 \\
 &= (x+6)(x+6) - 36 - 10 \\
 &= (x+6)^2 - 46
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad x^2 - 8x + 14 &= \left(x^2 - 8x + \frac{16}{1} \right) - \frac{16}{1} + 14 \\
 &= (x-4)(x-4) - 16 + 14 \\
 &= (x-4)^2 - 2
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad x^2 + x - 1 &= \left(x^2 + x + \frac{1}{4} \right) - \frac{1}{4} - 1 \\
 &= \left(x + \frac{1}{2} \right) \left(x + \frac{1}{2} \right) - \frac{1}{4} - 1 \cdot \frac{4}{4} \\
 &= \left(x + \frac{1}{2} \right)^2 - \frac{5}{4}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad x^2 - 100x + 100 &= \left(x^2 - 100x + \frac{2500}{1} \right) - \frac{2500}{1} + 100 \\
 &= (x-50)(x-50) - 2500 + 100 \\
 &= (x-50)^2 - 2400
 \end{aligned}$$

50
 $\times 50$
 00
 2500

$$\begin{aligned}
 (5) \quad x^2 + 7x + \frac{1}{2} &= \left(x^2 + 7x + \frac{49}{4} \right) - \frac{49}{4} + \frac{1}{2} \\
 &= \left(x + \frac{7}{2} \right) \left(x + \frac{7}{2} \right) - \frac{49}{4} + \frac{1}{2} \cdot \frac{2}{2} \\
 &= \left(x + \frac{7}{2} \right)^2 - \frac{47}{4}
 \end{aligned}$$

$$\begin{array}{r}
 7 \\
 \times 7 \\
 \hline
 49
 \end{array}$$

$$(6) \quad x^2 - \frac{x}{3} + 1$$

$$\begin{aligned}
 (7) \quad 2x^2 - 16x + 8 &= 2 \left[x^2 - \del{8} 8x + 4 \right] \\
 &= 2 \left[\left(x^2 - 8x + \frac{16}{1} \right) - \frac{16}{1} + 4 \right] \\
 &= 2 \left[(x-4)(x-4) - 16 + 4 \right] \\
 &= 2 \left[(x-4)^2 - 12 \right] \\
 &= 2 \cdot (x-4)^2 - 24
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad 5x^2 + 10x + 15 &= 5 \left[x^2 + 2x + 3 \right] \\
 &= 5 \left[x^2 + 2x + \frac{1}{1} - \frac{1}{1} + 3 \right] \\
 &= 5 \left[(x+1)(x+1) - \frac{1}{1} + 3 \right] \\
 &= 5(x+1)^2 - 5 \cdot 2 = 5(x+1)^2 - 10
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad 2x^2 + 5x + 1 &= 2 \left[x^2 + \frac{5}{2}x + \frac{1}{2} \right] \\
 &= 2 \left[x^2 + \frac{5}{2}x + \frac{25}{16} - \frac{25}{16} + \frac{1}{2} \right] \\
 &= 2 \left[\left(x + \frac{5}{4} \right) \left(x + \frac{5}{4} \right) - \frac{25}{16} + \frac{1}{2} \cdot \frac{16}{16} \right] \\
 &= 2 \left[\left(x + \frac{5}{4} \right)^2 - \frac{9}{16} \right] \\
 &= 2 \left(x + \frac{5}{4} \right)^2 - \frac{9}{8}
 \end{aligned}$$

$$(10) \quad 3x^2 + 3x + 7$$

$$(11) \quad x^2 + x + 1$$

$$(12) \quad 2x^2 + x + 1$$

$$(13) 10x^2 + 100x + 1000$$

$$(14) 3x^2 + 4x + 5$$